

About the Existence and Uniqueness of the Tricomi Issue Solution for a Special Type Equation

Goymatova Dilafruz Gofurjonovna

Andijan Institute of Mechanical Engineering, Assistent of the Department "Economy"

Article Information

Received: March 20, 2023

Accepted: April 21, 2023

Published: May 22, 2023

Keywords: lines of degeneracy, equation of mixed type, reguylar solution, and boundary value problem.

ABSTRACT

This article discusses the problems of the local boundary value problem for elliptical-parabolic equation of mixed type with different orders of degeneracy.

The article describes the uniqueness of the local boundary problem solution for a mixed elliptic-parabolic type equation with different order perturbation lines as

$$0 = \begin{cases} y^{m_1} u_x - x^{n_1} u_y, & x > 0 \\ y^{m_2} u_x + (-x)^{n_2} u_y, & x < 0 \end{cases} \quad (1)$$

we look at the equation in the field $D \subset R^2$. Where D is the area $x > 0$ at $u=0$, $x=H_1$, $y=h_2$ at OA , AV , VV_0 , intersections of straight lines and $x < 0$

$$\sigma: \frac{1}{q_2^2} (-x)^{2q_2} \frac{1}{p_2^2} y^{2p_2} = 1$$

the smooth line $y=0$ is bounded by the $A0O$ cross section of the straight line, $0 < m_1 < m_2 + 1$, $n_i = \text{const} (i=1,2)$, $h_1 = (2q_1)^{1/q_2}$, $h_2 = p_2^{1/p_2}$, $h = q_2^{1/q_2}$, $2q_1 = n_1 + 2$, $2q_2 = n_2 + 2$, $2p_2 = m_2 + 2$.

(1) the equation applies to the parabolic type at $x > 0$, $y > 0$, and to the elliptic type at $x < 0$, $y > 0$.

It is known that for Equation (1) in the infinite field, local and nonlocal boundary issues [1], [2] are studied in the works.

Let's enter the mark:

$$D_1 = \{(x, y) : 0 < x < h_1, 0 < y < h_2\}, D_2 = \{(x, y) : -h < x < 0, 0 < y < h_2\}.$$

Satisfying the following conditions, $U(x, y)$ is called a regular oriclassic solution of the function (1) equation.

- 1) $u(x, y) \in C^{2,1}(D_1) \cap C^2(D_2)$;
- 2) $u(x, y)$ satisfies the equation in fields D_1 and D_2 .

Tikomi issue. Define a function $U(x, y)$ satisfying these conditions:

- 1) $u(x, y) \in C(\bar{D}) \cap C^1(D)$;
- 2) $u(x, y)$ the regular solution of the equation in fields D_1 and D_2 is;
- 3) $U(x, y)$ satisfies the following conditions:

$$\begin{aligned} u(x, y) \Big|_{\sigma} &= \psi(x, y), (x, y) \in \sigma, \\ u_y \Big|_{y=0} &= v_0(x), -h < x < 0, \\ u \Big|_{y=0} &= \varphi_1(x), 0 \leq x \leq h_1, \\ u \Big|_{y=h} &= \varphi_2(u), 0 < u < h_2, \end{aligned}$$

where the given $\psi(x, y)$, $v_0(x)$, $\varphi_1(x)$, $\varphi_2(u)$, are the functions,

$$\begin{aligned} v_0(x) &\in C^1(-h < x < 0) \\ \varphi_1(x) &\in C(0 \leq x \leq h_1) \cap C^2(0 < x < h_1) \\ \varphi_2(u) &\in C(0 \leq y \leq h_2) \cap C^1(0 < y < h_2) \end{aligned}$$

$\psi(x, y)$ and the function $\psi(x, y) = xy \psi(x, y)$, $\psi(x, y) \in S(\sigma)$.

The following theorem is valid.

Theorem (extremum principle). The solution to the Triкоми problem, or domain D , is achievable at its positive maximum and negative minimum $(\sigma) \cup A_0O \cup (OA) \cup (AV)$.

Directly from the extremum principle comes the uniqueness of the Triкоми Question Solution for Equation (1).

It is known that the equation being considered with respect to an unknown function is called a functional equation. For example, expressing the property that a univariate function has a pair, a tangle, an excitable point, this

$$f(x) = f(-x), f(x) = -f(x), f \circ f(x) = x$$

peers consist of functional equations. The functional equation can be one or more variables. The following equations $f(x) = f(x/2)$, $f(x) = \cos 1/2 f(x/2)$, $f(0) = 1$ (f is a continuous function) are examples of one-variable functional equations.

Two variables $f(x+y) = f(x) + f(y)$, $f(x+y) = f(x)f(y)$:

$$f(x) = f(x) + f(y), f(x) = f(x)f(y)$$

functional equations are called Cauchy equations. At the same time we can cite the equation $f((x+y)/2) = (f(x)+f(y))/2$ - Iynsey and the equation $f(x+y)+f(x-y)=2f(x)f(y)$ -Dalamber. Typically, fuchsian equations have many solutions. Often the direct solution of the equation presents some difficulty. But it is possible to solve the functional equation based on some characteristic properties of the function being sought.

If the series are given by a recurrent Fourier in the form of functional equations, we show in examples that the nonmalum coyefficients of such series can be determined using the solution of the functional equation.

Example 1. This function $F(z) = (1-qz) (1-q^2 z) (1-q^3 z) \dots (|q| < 1)$ is known to extend to the rank row in the Form $F(z) = A_0 + a_1z + a_2z^2 + a_3z^3 + \dots$. Define the A_n -coyefficients of the

series shown using the functional equation $F(z)=(1-qz) F(qz)$. **Undo. In order of citation** $F(z) = (1 - qz) (1 - q^2z) (1 - q^3z) (1 - q^4z) \dots (1 - q^nz) \dots = A_0 + A_1qz + A_2q^2z^2 + A_3q^3z^3 + \dots$

And $(1-qz) F(qz) = A_0 + A_1qz + A_2q^2z^2 + A_3q^3z^3 + \dots - (A_0qz + A_1q^2z^2 + A_2q^3z^3 + A_3q^4z^4 + \dots)$
 $= A_0 + (A_1 - A_0)qz + (A_2 - A_1)q^2z^2 + (A_3 - A_2)q^3z^3 + \dots$

since is the functional equation given by $A_0 + (A_0 - A_1)qz + (A_2 - A_1)q^2z^2 + (A_3 - A_2)q^3z^3 + \dots = A_0 + A_1z + A_2z^2 + A_3z^3 + \dots$ gives a mirror in appearance. Equating the uniform vfracjal coefficients of z

$$(A_n - A_{n-1})qn = A_n, n=1,2,3,\dots$$

gives equality. From this equality

$$A_n = \frac{q^n}{q^n - 1} A_{n-1}, n = 1,2,3, \dots$$

let the relationship come. Finally, we define the coyefficients that are being sought from the last equality:

$$A_n = \frac{q^{\frac{n(n+1)}{2}}}{(q - 1)(q^2 - 1) \dots (q^n - 1)}, n = 1,2,3, \dots$$

Example 2. It is known that the inverse to

$F(z)=(1-qz) (1-q^2 z) (1-q^3 z) \dots(|q|<1)$ is $1/(F(z))=B_0+B_1 z+B_2 z^2+B_3 z^3 + \dots$

visually spread out in a row.

$$F(z) = (1-qz) F(qz)$$

define the B_n coyefficients shown using the functional equation.

Solution. In order of citation $\frac{1}{F(z)} = B_0 + B_1qz + B_2q^2z^2 + B_3q^3z^3 + \dots$

since the functional equation

$$\frac{1}{F(qz)} = \frac{1 - qz}{F(z)}$$

taking advantage of the fact that it looks,

$$B_0 + B_1qz + B_2q^2z^2 + B_3q^3z^3 + \dots = B_0 + B_1z + B_2z^2 + B_3z^3 + \dots - (B_0qz + B_1q^2z^2 + B_2q^3z^3 + B_3q^4z^4 + \dots) = B_0 + (B_1 - B_0q)z + (B_2 - B_1q)z^2 + (B_3 - B_2q)z^3 + \dots = B_0 + B_1qz + B_2q^2z^2 + B_3q^3z^3 + \dots$$

we write equality. Hence the given functional equation

$$B_0 + (B_1 - B_0q)z + (B_2 - B_1q)z^2 + (B_3 - B_2q)z^3 + \dots = B_0 + B_1qz + B_2q^2z^2 + B_3q^3z^3 + \dots$$

gives a mirror in appearance. Equating the koyefficients before the same levels of z gives the Equalities.

$$(B_n - B_{n-1}Q) = B_nqn, n=1,2,3,\dots$$

From these equalities $B_n = \frac{q}{1-q^n} B_{n-1}, n = 1,2,3, \dots$

Finally seeking from the last equalities

$$A_n = \frac{q^n}{(1 - q)(1 - q^2) \dots (1 - q^n)}, n = 1,2,3, \dots$$

we define coefficients

Conclusion. For special types of equations, the specificity of the Tricomi problem is related to the shape and complexity of the problem. When the issue is complex, solving it will make the Tricomi issue even more difficult to apply. In addition, the Tricomi question is a widely used method for finding a solution and is important in finding a solution for special types of equations.

Bibliography:

1. Bellman R., Cook K. *Differensialniye raznosti uravneniY.* – M.: Mir, 1961.
2. Salakhitdinov M.S., Krayeviye zadachi dlya uravneniya elliptico-parabolicheskogo tipa s razlichnimi poryadkami virozhdeniya. // *Dokladi AN RUz*, 1992. - №12. - S. 3-5.
3. Yefimov N.V. *Visshaya geometry.* – M.: Nauka, 1961.
4. Pauline E. Sege YE. *Zadachi theorem I analiza. Chast 1.* – M.: Nauka, 1978.
5. Болтабоев, М. (2022). Религиозные реформы, проведенные при соляном режиме. *Актуальные проблемы истории Узбекистана*, 1(1), 455-461.
6. Raxmonov, E. K. o'g'li, Qobilov, F. S. o'g'li, & Berdimuradov, X. T. o'g'li. (2023). RESPUBLIKAMIZDA YETISHTIRILAYOTGAN BUG'DOY DONLARINING FIZIK-KIMYOVIY KO'RSATKICHLARINING TAHLILI. *ILMIY TADQIQOT VA INNOVATSIYA*, 2(2), 95–101. Retrieved from <http://ilmiytadqiqot.uz/index.php/iti/article/view/144>
7. Qobilov, F. S. o'g'li, & Raxmonov, E. K. o'g'li. (2023). NON MAHSULOTLARINI TAYYORLASHDA QURUQ KLEYKOVINADAN QO'SHIMCHA SIFATIDA FOYDALANISH. *ILMIY TADQIQOT VA INNOVATSIYA*, 2(2), 58–63. Retrieved from <http://ilmiytadqiqot.uz/index.php/iti/article/view/139>
8. Sattorova, K. A. qizi, & Raxmonov, E. K. o'g'li. (2022). NON MAHSULOTLARINI SIFATINI OSHIRISHDA QO'LLANILADIGAN QO'SHIMCHALAR. *INTERNATIONAL CONFERENCES*, 1(1), 29–31. Retrieved from <https://researchedu.org/index.php/cf/article/view/230>
9. Комилов, А., & Болтабоев, М. (2022). ЎЗБЕКИСТОН–УМУМИЙ УЙИМИЗ. *Central Asian Academic Journal of Scientific Research*, 2(3), 141-148.
10. Болтабоев, М. (2023). СОВЕТ ХУКУМАТИНИНГ МАДРАСАЛАРГА БЎЛГАН МУНОСАБАТИ ТАРИХИ. *IJTIMOIY FANLARDA INNOVASIYA ONLAYN ILMIY JURNALI*, 3(3), 115-119.
11. Akhmedjanova, F. D. (2022). THE INFLUENCE OF LEARNING STYLES ON LANGUAGE TEACHING AT SECONDARY SCHOOL. *Academic research in educational sciences*, 3(3), 5-9.
12. Djavairovna, A. F. (2022). The Impact of Learning Styles in Teaching English.
13. O'G'Li, X. T. X., Berdimuradov, E. K. O. G. L., BUG'DOY, R. N. U. T., & ASOSLASH, N. T. V. *CARJIS*. 2022. № 10. URL: <https://cyberleninka.ru/article/n/navli-un-tortishda-bug-doy-navlarini-tanlash-va-asoslash> (дата обращения: 29.03. 2023).
14. Akhmedjanova, Farida Djavairovna. "The role and influence of feedback at foreign language learning achievement." *Science and Education 2.Special Issue 2 (2021): 89-93.*
15. Silberstein R. Solution of the equation $f(x)=f(1/x)$. // *Pill. Mag.* 7th Series, 30 (1940), p. 185-186.