

Study of Bending Vibrations of a Column in Rock

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Article Information

Received: December 05, 2022

Accepted: January 06, 2023

Published: February 07, 2023

Keywords: dispersion equation, casing, empirical method, monochromatic, monotonous, pipeline.

ABSTRACT

The propagation of stationary flexural elastic oscillations of drill strings and pipelines in rocks is studied, taking into account the viscous and elastic reactions of the rock. The dispersion equation is derived and analyzed. It is shown that there is a packet of waves in a rather narrow frequency range, which is characterized by the least attenuation and which is mainly captured by a sensor located farther from the oscillation source.

The direct empirical method [1], despite its generality and simplicity, has one very significant drawback. The fact is that the value of a and H , as well as the physical properties of the rock and casing in each particular case will, as a rule, be different, therefore, the systems under consideration and their additional functions will also be different. Therefore, the direct empirical method for its application requires a lot of preliminary work, significant economic and time costs.

It is much more expedient to use the direct empirical method only at the final stages of the study for selective verification of some approximate physical theory.

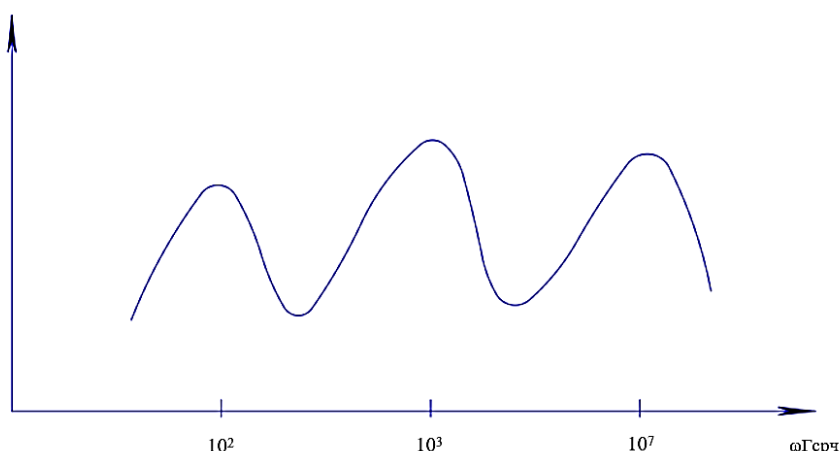


Figure. 1.

Let us turn to the estimation of the characteristic wavelength of the acoustic signal from the bit

[2]. This length turned out to be on the order of the casing diameter. This circumstance allows us to justify the following physical assumption: the casing string serves as a waveguide only for transverse bending vibrations that propagate along it as along a rod (beam). In other words, it is considered that the wave energy of other types of oscillations, perceived by the sensor, is negligible. Recall that there are quite a lot of these other types of oscillations in the system: direct longitudinal and transverse, surface, contact, longitudinal and torsional, etc.

Let $\omega(z,t)$ mean the displacement of the points of the neutral axis of the beam in the plane of transverse bending, z is the coordinate along the axis of symmetry of the beam, t is the time. The plane of the bend, obviously, pass through the axis of symmetry of the casing and through the point where the vibration source is located. As is known from the strength of materials, the beam equilibrium equation has the form [3]

$$E_0 J \frac{\partial^4 \omega}{\partial z^4} = q(z, t) \quad (1)$$

Here E_0 is the Young's modulus of the beam material, J is the moment of inertia of the beam cross section, q is the external load per unit length of the beam. The value of $E_0 J$ is called the bending stiffness of the rod. If the cross sections of the casing string can be modeled as a hollow cylinder, the value of J is equal to

$$J = \int_{r_1}^{r_2} \int_0^{2\pi} r^3 dr dQ = \frac{\pi}{2} (r_2^4 - r_1^4) \quad (2)$$

where r_1 and r_2 are the inner and outer radii of the hollow circular cylindrical pipe.

We define the external load $q(z,t)$ as follows

$$q(z,t) = a \frac{\partial^2 \omega}{\partial t^2} - K_\lambda \omega - \tau \frac{\partial \omega}{\partial t} \quad (3)$$

where, a , K_λ and τ are some physical constants of our system.

The first term in (3) is the mass force of inertia of the rod; coefficient α is equal to

$$\alpha = \rho F_S \quad (4)$$

where ρ is the density of the rod material, F_S is the cross-sectional area of the rod. In the case of a hollow cylinder, we have

$$F_S = \pi (r_2^2 - r_1^2) \quad (5)$$

The second term in (3) is the elastic reaction of the rock (approximation of Wickler's beam theories); K_λ is the so-called bed coefficient of the Wikler theories.

The third term in equation (3) takes into account all the main types of reactions that are essential for small fluctuations. According to (1) and (3), we find the following basic equation describing the propagation of bending waves along the casing string.

$$E_0 J \frac{\partial^4 \omega}{\partial z^4} = -\alpha \frac{\partial^2 \omega}{\partial t^2} - \tau \frac{\partial \omega}{\partial t} - K_\lambda \omega \quad (6)$$

Study of an infinite waveguide. Let us first study the propagation of waves in a rod of infinite length. Let us find the dispersion equation. We are looking for a solution to equation (6) in the form of a monochromatic stationary wave

$$\omega = \omega_0 \cdot e^{i(\lambda z - \omega t)} \quad (7)$$

where λ is the wavenumber, ω is the frequency, ω_0 is the amplitude

Substituting (7) into (6), we obtain the following relation relating the wavenumber to the frequency (dispersion equation)

$$\lambda^4 = \frac{1}{E_0 J} (-K_\lambda + \alpha \omega^2 + i \tau \omega) \quad (8)$$

A relation of this kind is called a dispersion equation [3]. It plays a major role in the analysis of the spectral characteristics of a linear filter [4].

It is convenient to write this relation as

$$\lambda = \frac{\left((-K_\lambda + \alpha \omega^2)^2 + \tau^2 \omega^2 \right)^{1/8}}{(E_0 J)^{1/4}} * (\cos \varphi + i \sin \varphi) \quad (9)$$

where

$$\varphi = \frac{1}{4} \operatorname{arctg} \frac{\tau \omega}{\alpha \omega^2 - K_\lambda}$$

Thus, stationary monochromatic waves can propagate through the casing only under the following conditions

$$\tau=0, \quad \omega^2 > K_\lambda / \alpha \quad (10)$$

Under these conditions, equation (9) will be as follows

$$\lambda = (E_0 J)^{-1/4} \cdot (K_\lambda + \alpha \omega^2)^{1/4} \quad (11)$$

Dependence graph $\lambda(\omega)$, according to formula (11), is shown in Figure 2.

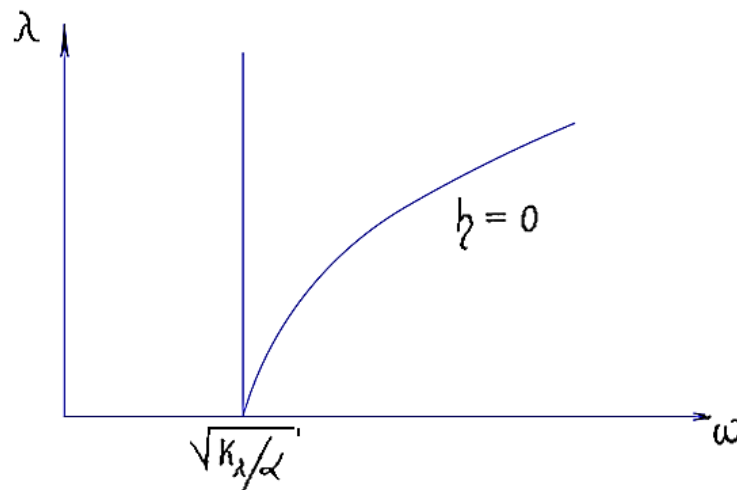


Figure. 2.

If condition (10) is not satisfied, then the wave number will always be complex. In this case, according to (7), the imaginary part λ will lead to exponential decay or exponential increase depending on the sign of $J_m \lambda$ perturbation at $z \rightarrow \infty$. The case of an increase in the perturbation should be discarded due to energy considerations; the case of a decrease remains.

Now we note that any perturbation on the string (for example, from drilling another well) can be represented as a package of given monochromatic waves of type (7) with different amplitudes.

Thus, we get the following important conclusions:

- a) any reaction of rock and aggregate always results in an exponential decrease in z of any wave disturbance in the casing string;
- b) the elastic reaction of the rock leads to exponential damping in z (in the absence of a viscous reaction) of those low-frequency oscillations in the perturbation package, cutting off these low frequencies with a sufficiently large length of the waveguide).

The presence of a viscous reaction at a sufficiently large length of the waveguide (casing string) leads to the damping of any packet of disturbances, i.e. to the fact that the signal does not pass through our system, tk. $Z/a \gg 1$ in all practically important cases. Fortunately, the viscous reaction is always much less than the elastic one; therefore, this damping effect in all practical cases is of a secondary, albeit fundamental, nature. So, we will assume that

$$|\tau\omega| \ll |\alpha\omega^2 - K_\lambda| \quad (12)$$

In this case, the conditions formula (9) will take the following form

$$\lambda = \left(\frac{\alpha\omega^2 - K_\lambda}{E_0 J} \right) * \left(1 + \frac{i\tau\omega}{4(\alpha\omega^2 - K_\lambda)} \right) \quad (13)$$

Substituting (13) into (7) we come to the following conclusion: the presence of a small viscous reaction leads to the damping of any signal in our system according to the following law

$$|\omega| = \exp \left(- \frac{i\tau Z}{4(E_0 J)^{1/4} (\alpha\omega^2 - K_\lambda)^{3/4}} \right) \quad (14)$$

The coefficient of proportionality in this formula depends on the intensity of the initial perturbation. Exploring the function

$$f(\omega) = - \frac{\omega}{(\alpha\omega^2 - K_\lambda)^{3/4}} \quad \text{at } \omega^2 > K_\lambda/\alpha \quad (15)$$

The derivative of this function, equal to

$$f'(\omega) = - \frac{\alpha\omega^2 + 2K_\lambda}{2(\alpha\omega^2 - K_\lambda)^{7/4}} \quad (16)$$

is always negative. Therefore, $f(\omega)$ kills monotonically; the graph of the function $f(\omega)$ has the form shown in Fig. 3. The analysis of this function and the law (14) shows that the attenuation different harmonics occur differently. Frequencies close to

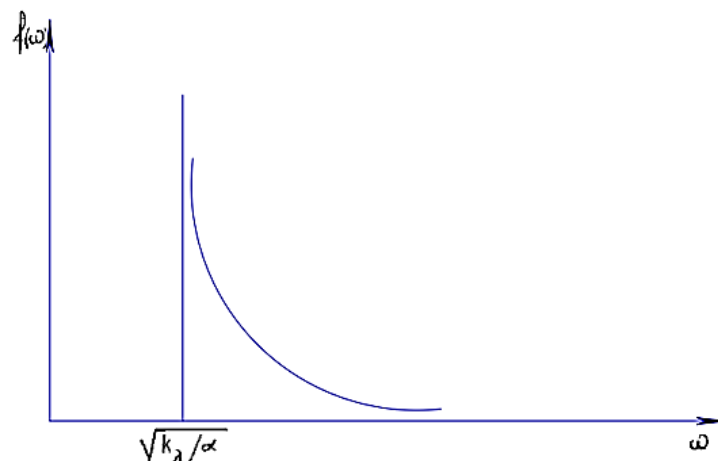


Figure. 3.

$$\omega_* = \sqrt{K_\lambda / \alpha} \quad (17)$$

decays very quickly even for a very small viscous reaction; very high frequencies are attenuated very weakly. However, it should be emphasized that this conclusion is connected with the assumptions about the independence of the coefficient τ from the frequency ω . In fact, τ depends on ω (the greater the acceleration $\partial \omega^2 / \partial t^2$, the greater the viscous reaction); assuming, for example, that $\tau = \tau_0 + \tau_0 \omega^2$, we obtain the following function of ω , which is in the exponent of the law (14)

$$f(\omega) = \frac{\tau_0 \omega + \omega^3 \tau_2}{(\alpha \omega^2 - K_\lambda)^{3/4}}, \quad (\omega^2 > K_\lambda / \alpha) \quad (18)$$

The derivative of this function, equal to

$$f'(\omega) = \frac{\alpha \tau_2 \omega^4 - (\omega \tau_2 K_\lambda + \alpha \tau_0) \omega^2 - 2 \tau_0 R_\lambda}{2(\alpha \omega^2 - K_\lambda)^{5/4}} \quad (19)$$

Has one zero at $\omega > \omega_*$. The graph of the function $f(\omega)$, according to (18), has the form shown in Fig. 4. Note the following properties of this function

$$\text{at } \omega \rightarrow \omega_* \quad f(\omega) = \frac{\omega_*^{1/4} (\tau_0 + \tau_2 \omega_*^2)}{(2\alpha)^{3/4} (\omega - \omega_*)^{3/4}} \rightarrow \infty$$

$$\text{at } \omega \rightarrow \infty \quad f(\omega) = \frac{\tau_2}{\alpha^{3/4}} \omega^{3/2} \rightarrow \infty \quad (20)$$

$$\text{at } \omega \rightarrow \omega_0 \quad f(\omega) = 0 \quad (\text{minimum function}).$$

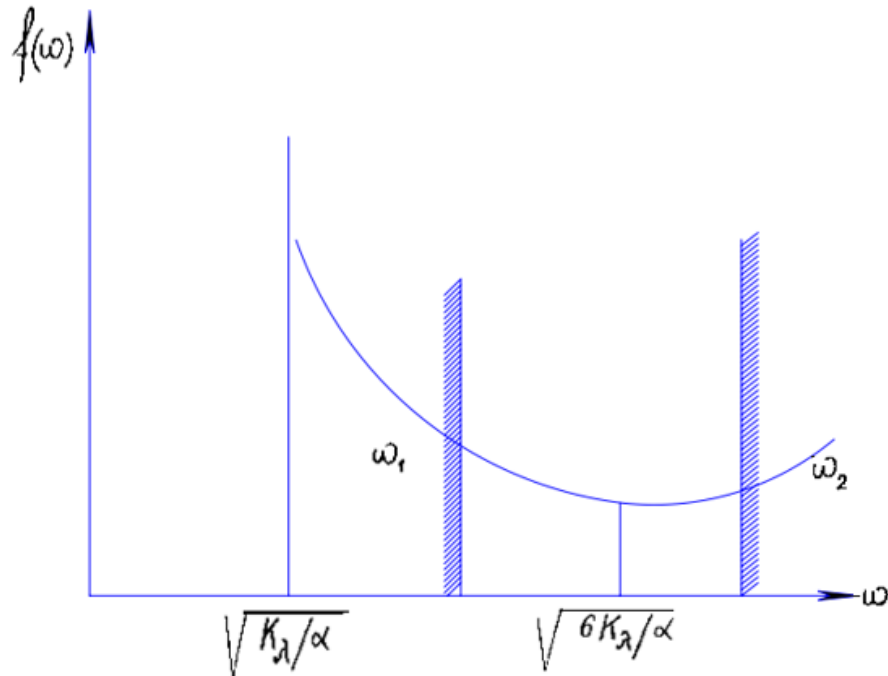


Figure. 4.

So there is a rather weak dependence of the coefficient on the part ω , i.e.

$$\eta_0 \gg \eta^2 \omega^2 \quad (21)$$

then from here you can find the values of the minimum of the function at the point

$$\alpha \eta_2 \omega_0^2 - 6 \eta_2 K_\lambda = 0$$

those.

$$\omega_0^2 = 6 \frac{K_\lambda}{\alpha} \quad (22)$$

Hence, comparing with (17), we obtain

$$\omega_0 = \sqrt{6} \omega_* \quad (23)$$

$$f(\omega_0) = \frac{(\eta_0 + 6 \eta_2 \omega_*^2) \sqrt{6} \omega_*}{\alpha^{3/4} \circ 5 \omega_*^2} \quad (24)$$

Harmonics with frequency ω_0 have the weakest attenuation. High-frequency oscillations and oscillations with a frequency close to ω^* decay very quickly even for very small viscous effects

Inside a certain region (ω_1, ω_2) , where $\omega_1 > \omega_*$, $\omega_0 < \omega_2 < \infty$, damping can practically be neglected and formula (11) can be used.

The results of the study of the throughput of our system can be formulated as follows:

1. The casing is a waveguide filter capable of passing signal harmonics with a frequency in the range (ω_1, ω_2) .
2. Values ω_1 and ω_2 , which define these boundaries, depend on the sensitivity of the sensor and measuring equipment, on the signal intensity and the length of the casing string H/a (distance from the sensor to the bit); in any case, $\omega_1 > \omega_*$, where ω_* is determined by formula (17), and $\omega_2 > \omega_0$, where ω_0 is determined by formula (23)
3. A packet of waves (signal) with frequencies in the range ω_1, ω_2 is subjected to dispersion, determined by the law (11); the phase and group velocities of the packet can be easily found from here [5-7]:

phase velocity

$$V_p = \frac{\omega}{\lambda} = \left(\frac{E_0 J}{\alpha} \right)^{1/4} \omega (\omega^2 - \omega_*^2)^{-1/4} \quad (25)$$

$$V_g = \frac{d\omega}{d\lambda} = \left(\frac{E_0 J}{\alpha} \right) \circ \frac{2}{\omega} \omega (\omega^2 - \omega_*^2)^{3/4} \quad (26)$$

As you can see, the group velocity is a monotonically increasing function of ω in the range (ω_1, ω_2) . The phase velocity decreases in the range (ω_1, ω_{10}) and increases in the range (ω_{10}, ω_2) ; the value of ω_{10} is equal to $\sqrt{2} \omega_*$.

To accurately determine ω_1 and ω_2 , it is necessary to solve the boundary value problem for equation (6) with given initial perturbations.

As an example, consider the propagation of the initial delta-shaped pulse-packet, the expansion of which contains harmonics of all frequencies ω (from $-\infty$ to $+\infty$) of the same amplitude [8].

Let us first recall the Heisenberg uncertainty principle

$$\Delta \omega \cdot \Delta t = 2\pi \quad (27)$$

Here t is the characteristic width of the pulse (wave packet), ω is the characteristic width of the frequency range present in this pulse. For a delta-shaped pulse it will be $\Delta t \rightarrow 0$, $\Delta \omega \rightarrow 0$.

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