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Application of the Theory of Linear Differential Equations to the Study of Some Oscillations

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ABSTRACT

The article describes the theory of linear differential equation for the problems of some oscillation moving bodies.

In this paper, small oscillations of a flat mathematical pendulum and oscillations of a load suspended on a vertical spring are considered. Linear differential equations are solved. The solutions of the obtained homogeneous and non-homogeneous differential equations completely give the approximate values of the pendulum deviation as a function of time, i.e. if a spring with loads acts vertically directed force, the magnitude of which depends on time.

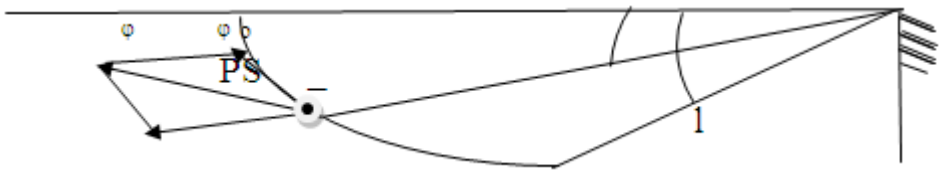
Let us consider two problems from the theory of oscillations, which are reduced to second-order linear differential equations with constant coefficients.

The problem of small oscillations of a plane mathematical pendulum.

Material point of mass ***m*** is suspended from an inextensible weightless thread of length *l.* If the thread with a load (pendulum) is deflected from the equilibrium position by an angle φ_0 and inform it of the initial speed v_0, so that the velocity vector $\boldsymbol{v_0}$ lies in a plane passing through the new position of the thread and the equilibrium position, then the pendulum begins to oscillate, remaining in the specified plane. In this case, the position of the pendulum can be characterized by the value of the angle φ, showing the deviation of the pendulum from the equilibrium position. Let's find the law of motion of the pendulum, that is, we will find the law that allows us to determine the position of the pendulum at any time t.

Under the conditions of the problem, a material point *m* moves along an arc of a circle of radius ***l*** centered at the point of suspension of the thread. Let us denote by ***s the*** magnitude of the arc of this circle corresponding to the angle φ: $s = l\varphi$ (Fig. 1).

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At every moment of time *t* point m is acted upon by gravity $P = mg$ (its tangential component is equal in magnitude to $mg\ sin\varphi$ and directed in the direction of decreasing $|s|$), the thread tension force *T* directed along the thread from point *m* to the thread suspension point (its tangential component is zero) and the resistance force of the medium directed tangentially in the direction opposite to the velocity vector and proportional in magnitude to this speed, i.e. equal to $\mu\frac{ds}{dt}$,where μ - coefficient of proportionality, $\mu>0$. Based on Newton's second law, we obtain the differential equation of motion of the pendulum:

$$m\frac{d^2s}{dt^2} = -mg\ sin\varphi - \mu\frac{ds}{dt}.$$

Substituting the value of *s* ($s = l\ \phi$) and denoting

$$\sqrt{\frac{g}{l}} = \omega, \qquad \frac{\mu}{m} = 2\lambda,$$

we write the equation of motion of the pendulum in the form:

$$\frac{d^2\varphi}{dt^2} + 2\lambda\frac{d\varphi}{dt} + \omega^2 sin\varphi = 0. \qquad (1)$$

This differential equation is called ***the equation of free oscillations of the pendulum.***

If on a material point *m* an external perturbing force acts, the value of which depends on time, a free term appears in the equation of motion of the pendulum:

$$\frac{d^2\varphi}{dt^2} + 2\lambda\frac{d\varphi}{dt} + \omega^2 sin\varphi = f(t). \qquad (2)$$

This equation is called ***the equation of forced oscillations of the pendulum.*** Differential equations (1) and (2) are not linear, since the unknown function φ is under the sine sign. However, if we consider small fluctuations, then $sin\varphi$ can be replaced by φ.

The resulting equations

$$\frac{d^2\varphi}{dt^2} + 2\lambda\frac{d\varphi}{dt} + \omega^2\varphi = 0 \qquad (3)$$

and

$$\frac{d^2\varphi}{dt^2} + 2\lambda\frac{d\varphi}{dt} + \omega^2\varphi = f(t) \qquad (4)$$

are called ***differential equations of small oscillations of the pendulum.***

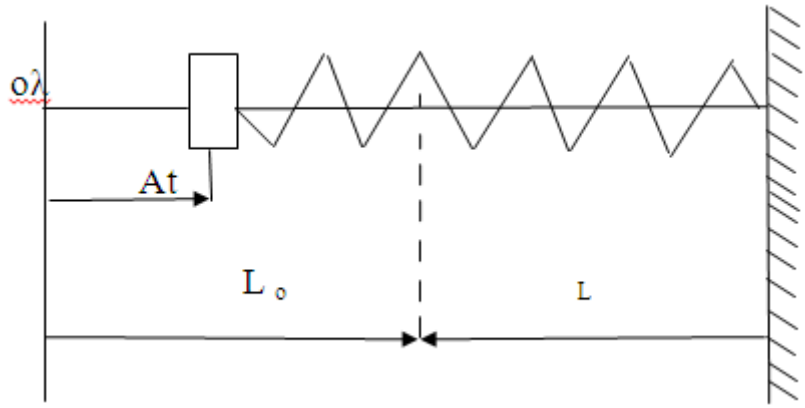
Solutions (3) and (4) will give inaccurate, but practically quite satisfactory approximate values of the pendulum deflection $\boldsymbol{\varphi}$ as a function of time *t* .

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The problem of oscillation of a load suspended on a vertical spring.

A weight $P$is suspended on a vertical spring, the length of which in the unloaded state is equal to $\boldsymbol{l}$. If the spring with the load is pulled or compressed (in the vertical direction) giving the load some initial speedv_0 , then it will begin to oscillate. Let's find out according to what law these fluctuations will occur, that is, we will find a formula expressing the deviation of the load from the equilibrium position (point *O*) at any time ***t*** , provided that the mass of the spring is not taken into account.

Let's direct the axis *OY* vertically top (from point *O* to the point of suspension of the spring). Let l_0 be the static elongation of the spring, that is, the distance from the end of the unstretched spring to the equilibrium position of the load **,** y_0 is the initial deviation, and ***y*** is the deviation of the load from the equilibrium position at time t (Fig. 2).



The force acting on the load is equal to the sum of the force of gravity of the load, directed in the negative direction and equal in magnitude to *P*, *the* resistance force of the medium, which is directed in the direction opposite to the movement of the load and is proportional in magnitude to the speed of movement $\frac{dy}{dt}$, and the tension force of the spring. The tension force of the spring is directed along the axis *OY* , its value, according to Hooke's law, is proportional to the elongation of the spring, that is, it is equal to $c(l_0 - y)$**,** where c is a constant coefficient of spring stiffness.

Based on Newton's second law, we obtain the equation for the movement of a load:

$$m\frac{d^2y}{dt^2} = c(l_0 - y) - P - \mu\frac{dy}{dt},$$

where$m = \frac{P}{g}$ - the mass of the cargo, μ- coefficient of proportionality, μ>0. Since in the equilibrium position of the load *(y \u003d 0*) the weight of the spring is balanced by the tension force, then

$$P = cl_0$$

and, consequently, the equation of the movement of the load will be written in the form:

$$m\frac{d^2y}{dt^2} = -cy - \mu\frac{dy}{dt}$$

or, if denoted

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$$\sqrt{\frac{c}{m}} = \omega, \qquad \frac{\mu}{m} = 2\lambda,$$

as:

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = 0. \qquad (5)$$

This equation is called ***the equation of free vibrations of a*** load suspended on a spring . If a spring with a load a vertically directed external force acts, the value of which depends on time (for example, a force that causes the suspension point of a spring to periodically oscillate), a free term will appear in the equation of motion of the load $f(t)$:

$$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = f(t). \qquad (6)$$

This linear inhomogeneous differential equation is called ***the equation of forced vibrations of the load.***

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